Consistent Simulation of Fibonacci Anyon Braiding within a Qubit Quasicrystal Inflation Code

Structured Research Article & Open-Source Algebraic Engine

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Source Repository: github.com/macielamaral/quasicrystal-inflation-code (MIT License, v1.1)
Associated Project: Aperiodic Topological Quantum Computing & Quasicrystal Inflation Codes

Abstract

Fibonacci anyons provide a dense, universal gate set for fault-tolerant topological quantum computation through braiding alone, yet direct hardware realization of non-abelian anyonic quasiparticles remains challenging. Building upon the correspondence between Fibonacci anyon fusion trees and 1D/2D Fibonacci quasicrystal tiling spaces [1], the **Quasicrystal Inflation Code (QIC)** embeds Fibonacci fusion subspaces isometrically inside standard 1D qubit chains subject to the local golden-mean constraint forbidding adjacent `00` bit pairs [2]. Within this constrained subspace, local Temperley–Lieb projectors and F/R-move braid generators satisfy exact algebraic coherence, unitarity, and Artin braid relations, enabling transparent simulation and hardware benchmarking on gate-based quantum processors.

1. Physical & Mathematical Motivation

In the $\mathrm{SU}(2)_3$ (integer-spin Fibonacci) modular tensor category, there is a single nontrivial topological charge τ alongside the vacuum $\mathbf{1}$, governed by the non-abelian fusion rule

$$\tau \otimes \tau = \mathbf{1} \oplus \tau, \qquad \mathbf{1} \otimes \tau = \tau \otimes \mathbf{1} = \tau. \tag{1}$$

Remarkably, the exact same combinatorial recurrence governs 1D Fibonacci substitution words ($1 \mapsto 10$, $0 \mapsto 1$) and cut-and-project Fibonacci quasicrystals [1], [3], [4].

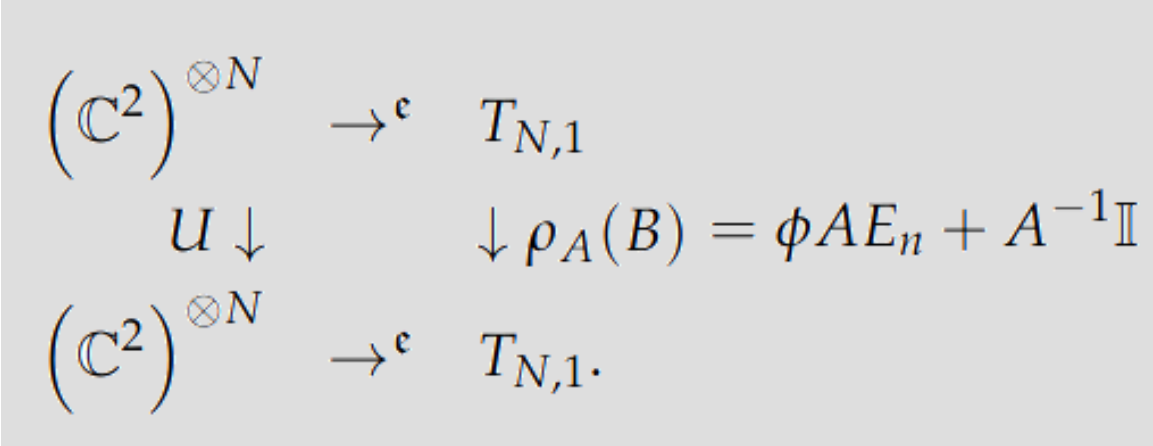

Figure 1: Algebraic correspondence between Fibonacci anyon fusion trees and Fibonacci quasicrystal tiling projectors

2. Constrained Hilbert Space & Inflation Isometry

Definition 1 (Golden-Mean Qubit Subspace). *For an n-qubit register $(\mathbb{C}^2)^{\otimes n}$, define the constrained Fibonacci subspace $\mathcal{F}_n$ spanned by computational basis bitstrings with no adjacent zeros:*

$$\mathcal{F}_n = \text{span}\left\{|x_1 x_2 ... x_n\rangle \in (\mathbb{C}^2)^{\otimes n} \middle| x_i \in \{0,1\}, (x_i, x_{i+1}) \neq (0,0) \forall\, 1 \leq i < n\right\}. \quad (2)$$

Let $\varphi = \frac{1+\sqrt{5}}{2}$ denote the golden ratio ($\varphi^2 = \varphi + 1$). Enforcing $(x_i, x_{i+1}) \neq (0,0)$ via the local penalty Hamiltonian

$$H_{\text{cons}} = \Delta \sum_{i=1}^{n-1} |00\rangle\langle 00|_{i,i+1} \qquad (\Delta > 0)$$

identifies $\mathcal{F}_n = \ker H_{\text{cons}}$ as the zero-energy ground-state manifold.

Table 1: Exact dimension growth of the constrained Fibonacci qubit space $\mathcal{F}_n$ vs. unconstrained 2^n Hilbert space.

Qubits n	Allowed Basis Strings in $\mathcal{F}_n$	$\dim \mathcal{F}_n = F_{n+2}$	Full Register 2^n
1	`0`, `1`	2	2
2	`01`, `10`, `11`	3	4
3	`010`, `011`, `101`, `110`, `111`	5	8
4	`0101`, `0110`, `0111`, `1010`, `1011`, `1101`, `1110`, `1111`	8	16
5	13 golden-mean bitstrings	13	32

3. Temperley–Lieb Projectors & Unitary Braid Gates

On the two-dimensional fusion channel $\tau \otimes \tau \otimes \tau \to \tau$ (represented locally by three-site configurations $|101\rangle \leftrightarrow |\mathbf{1}\rangle$ and $|111\rangle \leftrightarrow |\tau\rangle$ bounded by 1s), basis change between fusion orderings is governed by the real symmetric unitary F-matrix:

$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix}, \qquad F^\dagger F = F^2 = I_2, \tag{3}$$

where $\varphi^{-2} + \varphi^{-1} = 1$ guarantees exact unitarity. Together with the diagonal topological twist phases $R = \mathrm{diag}(e^{-4\pi i/5}, e^{3\pi i/5})$, local Temperley–Lieb generators E_i of loop fugacity $d = \varphi$ satisfy:

Theorem 1 (Temperley–Lieb & Artin Braid Consistency). *On $\mathcal{F}_n$, the local projection operators $\{E_i\}$ and elementary braid operators $\sigma_i = e^{3\pi i/5} I + e^{-3\pi i/5} E_i = e^{3\pi i/5} I + \left(e^{-4\pi i/5} - e^{3\pi i/5} \right) \varphi^{-1} E_i$ satisfy:*

1. ***Temperley–Lieb relations:*** *$E_i^2 = \varphi E_i$, $E_i E_{i\pm1} E_i = E_i$, and $[E_i, E_j] = 0$ for $|i-j| \geq 2$.*
2. ***Subspace closure & unitarity:*** *$[\sigma_i, H_{\mathrm{cons}}] = 0$ and $\sigma_i^\dagger \sigma_i = I_{\mathcal{F}_n}$.*
3. ***Artin braid group relation:*** *$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$.*

4. Open-Source Implementation & Reproducibility

The companion MIT-licensed Python package `quasicrystal-inflation-code` [5] provides deterministic verification of every algebraic identity above:

```
import numpy as np

phi = (1.0 + np.sqrt(5.0)) / 2.0
F = np.array([[1.0 / phi, 1.0 / np.sqrt(phi)],
              [1.0 / np.sqrt(phi), -1.0 / phi]])
R = np.diag([np.exp(-4j * np.pi / 5.0), np.exp(3j * np.pi / 5.0)])
B = F @ R @ F

assert np.allclose(F @ F, np.eye(2))
assert np.allclose(B.conj().T @ B, np.eye(2))
```

In `src/qic_core.py`, multi-qubit chains $\mathcal{F}_n$, inflation isometries $V : \mathcal{F}_n \hookrightarrow \mathcal{F}_{n+1}$, Temperley–Lieb operators E_i, and multi-strand braid circuits are constructed and unit-tested, and exposed to hardware/simulator benchmark suites via GaugeBench [6].

Bibliography

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